

## INFLUENCE OF THE MASS TRANSFER OF WATER IN CARTILAGINOUS TISSUE ON THE TEMPERATURE FIELD INDUCED BY LASER RADIATION

É. N. Sobol',<sup>a</sup> M. S. Kitai,<sup>a</sup>  
A. V. Zakharchenko,<sup>b</sup> Ya. M. Zhileikin,<sup>c</sup> and  
A. B. Kukarkin<sup>c</sup>

UDC 576.7:536.24

*A study is made of the influence of the mass transfer of water on the temperature field occurring in cartilaginous tissue under the action of laser pulses. Two different mechanism of mass transfer of water in cartilaginous tissue (molecular-diffusion mechanism and laminar flow) have been considered in solving heat- and mass-transfer equations. The calculations have shown that the maximum temperature is attained inside the sample because of the evaporation of water from the biological-tissue surface. The influence of different parameters of laser radiation and mass transfer of water on the surface temperature, on the maximum value of the temperature, on the position of the temperature maximum, and on the characteristic time at which the diffusion-limited relaxation of stresses in the cartilaginous tissue occurs has been analyzed.*

**Introduction.** In recent years, the attention of researchers has been attracted to the thermal nonablation action of lasers on biological tissues, for example, with the aim of relaxing stresses and changing the shape of cartilages ("shaping") [1]. Water is the main component of biological tissues (in cartilaginous tissue, its concentration amounts to about 80%); therefore, evaporation in laser heating influences not only its spatial distribution but also the temperature field. Whereas the influence of the evaporation effect on the temperature field has been investigated to a certain extent [2], the features associated with mass transfer and with the resulting deficiency of water in the surface layers of the laser-irradiated tissue have not been adequately studied.

In this work, we make a theoretical analysis of the temperature field induced by laser radiation with allowance for the mass transfer of water in cartilaginous tissue. The applied objective of the present investigation is to develop theoretical foundations of the medical procedure of changing the shape of cartilaginous tissue by the action of laser radiation. The relaxation of stresses in cartilaginous tissue occurs in short-duration laser heating to a temperature above  $T^* \sim 70^\circ\text{C}$  [1, 2]. If the heating of the tissue above  $T^*$  is not too strong or long, denaturation and destruction of biopolymers are absent (or insignificant) [1]. Therefore, in the present investigation, the emphasis will be on moderate intensities of laser heating for which the temperature is no higher than  $100^\circ\text{C}$  as a rule.

**Initial Equations and the Approximations Used.** Let us consider the processes of transfer of heat and mass in an irradiated plane-parallel plate on the basis of the equations of heat conduction and diffusion:

$$\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial x^2} + f(x, t), \quad (1)$$

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial x} \left[ D(T) \frac{\partial m}{\partial x} \right]. \quad (2)$$

Below we will investigate the molecular-diffusion mechanism of mass transfer for which

---

<sup>a</sup>Institute of Problems of Laser and Information Technologies, Russian Academy of Sciences, Troitsk, Russia; email: sobol@laser.ru; <sup>b</sup>Intersectoral Supercomputer Center, Moscow, Russia; <sup>c</sup>Scientific-Research Computation Center, M. V. Lomonosov Moscow State University, Moscow, Russia. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 76, No. 2, pp. 94–100, March–April, 2003. Original article submitted June 13, 2002; revision submitted September 9, 2002.

$$D = D(T) = D_0 \exp(-E_a/RT), \quad (3)$$

and the mechanism of laminar flow of water in the pores which is described by Darcy's law [3, 4]. It is common knowledge [5] that different mechanisms of mass transfer can be described in many cases by Eq. (2). For the laminar flow the temperature dependence of the diffusion coefficient is attributed to the temperature dependence of the viscosity of water (see below).

Equations (1) and (2) satisfy the initial and boundary conditions

$$T(x, 0) = T_0, \quad m(x, 0) = m_0; \quad (4)$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0, x=l} = \delta(x) m^{2/3} (vQ_v/\lambda) \exp[-(Q/RT(x, t))]; \quad (5)$$

$$\left. \frac{\partial m}{\partial x} \right|_{x=0, x=l} = (\lambda/Q_v D) \left. \frac{\partial T}{\partial x} \right|_{x=0, x=l}. \quad (6)$$

Boundary conditions (5) and (6) describe the influence of the effect of evaporation of water on the temperature field and the space-time distribution of the water. It is precisely allowance for this effect that differentiates the present work from the traditional consideration of the action of laser radiation on biotissues. See [6] for the derivation of boundary condition (6).

The formulation of the problem in the form (1)–(6) assumes the following approximations: 1) surface cooling of the sample is exclusively due to the evaporation of water; cooling by air (blowing) is assumed to be small and it is disregarded; 2) the diameter of a laser beam is much larger than all the remaining characteristic geometric dimensions of the problem (thickness of the sample, characteristic thickness of the layer where thermal absorption occurs), which enables us to confine ourselves to a one-dimensional approximation; 3) the coefficients of thermal diffusivity and heat capacity and the functional parameters of the mechanism of mass transfer are independent of the coordinates, time, and temperature; 4) the scattering of light in the tissue is disregarded and only its absorption by periodic laser pulses is allowed for; therefore, the function  $f(x, t)$  characterizing the heating of the sample by laser radiation has the form  $f(x, t) = (E\alpha/C_p) \exp(-\alpha x) \theta(t)$ , where  $\theta(t) = 1$  for  $0 \leq t \leq \tau_p$  and  $\theta(t) = 1$  for  $\tau_p \leq t \leq 1/f_r$ ; this function is prescribed on the segment  $[0, 1/f_r]$  and is extended with a period  $1/f_r$  to the region  $[0 \leq t \leq t_1]$ ; when  $t > t_1$ ,  $\theta(t) = 0$ ; the functions  $T(x, t)$  and  $m(x, t)$  are defined in the region  $[0 \leq x \leq l] \times [0 \leq t \leq t_0]$ ; 5) the mechanism of diffusion of water is independent of the "prehistory" of the sample and it remains constant in the process of laser action.

**Methods of Solution of the Equations.** Problem (1)–(6) was solved by numerical methods. To construct the computational model we used the principle of discretization of the initial problem, i.e., the continuous functions of state of the sample (temperature, concentration of water) were approximated by step quantities considered to be constant within each separate coordinate cell and stepwise changing in passage to the neighboring cell. The step of the coordinate grid was selected using software with such a condition that the relative value of the step of the function at the boundary of the coordinate cells does not exceed the value prescribed in advance (discreteness coefficient).

We considered independent processes of absorption of laser radiation by the sample, evaporation of water from the surface, heat transfer, and motion of water inside the sample. For the step functions of state these processes were described by difference equations with a step corresponding to that of the coordinate grid. In the case where the characteristics of the process at the boundary of two cells are influenced by the state of the latter (for example, in calculating the motion of water from one cell to the other in accordance with the diffusion law when the diffusion coefficient is independent of the temperature) we employed in the equations a value intermediate between the values in the neighboring cells.

When the initial conditions are fixed, the described system of difference equations has a single solution; one can find this solution by direct calculation, composing the tables of state of the sample for any instant of time accurate to a step [7, 8]. Check of the results obtained with the use of different values of the calculation step and the discreteness coefficient has shown that the solutions found are stable in relation to a variation in the discretization parameters and to small variations of the initial data.

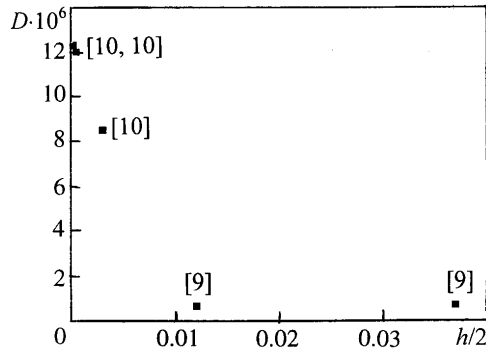


Fig. 1. Effective coefficients of diffusion of water  $D_{\text{ef}}$  vs. root-mean-square distance  $h/2$  traversed by the diffusing molecules. The figures in square brackets are the numbers of the references in which the corresponding data have been measured.

To check the exactness of the numerical solution of the system of equations and to test the program we used: 1) test calculations on a finer difference grid and with a larger number of iterations; 2) check of the fulfillment of the physical conservation laws associated with the fact that cooling is assumed to be due just to evaporation; the evaporation of one mole of water causes a decrease in the internal energy of the sample by  $Q$ . Therefore, for the time  $t > t_i$  (after switching off the laser) we computed the discrete analogs of the integrals

$$S(t) = \int_0^1 T(x, t) dx, \quad M(t) = \int_0^1 m(x, t) dx. \quad (7)$$

The change in these quantities satisfies the relation  $|\partial S/\partial M| = Q/C_p = \text{const}$ . The calculations showed that this equality is observed with an accuracy of  $\sim 1\%$ .

**Selection of the Design Parameters.** In solving the problem, one should differentiate two types of parameters: constant parameters (characterizing a given substance) and variable parameters the dependence on which is the subject of investigation. For cartilaginous tissue the constant parameters are  $\chi = 1.4 \cdot 10^{-3} \text{ cm}^2/\text{sec}$  [9],  $m_0 = 0.042 \text{ mole/cm}^3$  [2],  $\nu \approx 3 \cdot 10^{10} \text{ Hz}$  [2],  $Q = 44 \text{ kJ/mole}$  [9], and  $T_0 = 293 \text{ K}$ .

For the diffusion model of mass transfer of water the diffusion coefficient was measured directly [9, 10]. According to [9],  $D \approx 7.1 \cdot 10^{-7} \text{ cm}^2/\text{sec}$  at 300 K for the case where the Fourier factor is  $\text{Fo} = 4Dt/h^2 \geq 1$ . The diffusion coefficient in the cartilaginous tissue was computed using the NMR pulse-gradient technique (for a content of water of 77% by mass) [10]; different values of this quantity as functions of the measurement time  $\tau$  and the value of the parameter  $h/2 = (D\tau)^{1/2}$  were obtained. Thus, at room temperature we had  $D = 1.2 \cdot 10^{-5} \text{ cm}^2/\text{sec}$  for  $\tau \approx 13 \cdot 10^{-3} \text{ sec}$  and  $D = 1.0 \cdot 10^{-5} \text{ cm}^2/\text{sec}$  for  $\tau \approx 100 \text{ msec}$  [10]. The reasons for the dependence of  $D$  on  $h$  (Fig. 1) are the difference in the diffusion rates in different regions of the inhomogeneous cartilaginous tissue and the features of the measurement procedure employed. Using NMR, we determine the average rate of diffusion for the selected volume, and since we select a volume with a maximum signal (with a maximum value of  $D$ ) the diffusion coefficient turns out to be larger for small  $h$  and with increase in  $h$  the quantity  $D$  is averaged over the regions with high and low rates of diffusion. Since we are interested in mass-transfer distances of about 0.1 mm, in the calculations we employed the following data [9]:  $D_0 = 0.119 \text{ cm}^2/\text{sec}$  and  $E_a = 30 \text{ kJ/mole}$ .

In determining the diffusion coefficient for the model of laminar flow of water in cartilaginous tissue, the initial information was provided by the experimental measurements of the coefficient of its flow [11] through a plate of cartilaginous tissue under the action of the external pressure of water. The flow rate of water per unit time can be written in the form  $q = \Delta PA/(8\mu l)$ . In converting from  $q$  to  $D_{\text{ef}}$ , we employed the equality  $v_s^2 = (\partial P/\partial x)/(\rho_w \partial m/\partial x)$  [4] and the Nernst law  $q = -DA(\partial m/\partial x)$ , whence  $\partial m/\partial x = v_w^{-2}((\partial P/\partial x)/\rho_w)$ . The ratio  $q/(A(\partial P/\partial x))$  (at room temperature) has been measured experimentally in [11]. In combination with the above formulas, this enables us to determine  $D_{\text{ef}}$  for the laminar flow ( $D_{\text{ef}} = 3.2 \cdot 10^{-5} \text{ cm}^2/\text{sec}$  at  $T = 300 \text{ K}$ ). On model grounds we assume the Arrhenius depend-

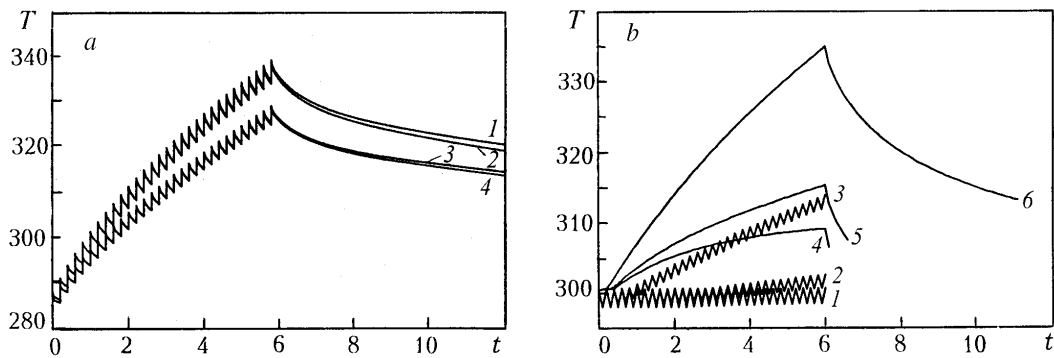


Fig. 2. Temperature of the irradiated surface vs. time for different mechanisms of mass transfer and irradiation parameters: a) ( $l = 1.2$  mm,  $f = 5$  Hz, and  $\alpha = 20$  cm<sup>-1</sup>): 1, 3) diffusion mechanism of mass transfer; 2, 4)  $m = m_0$ ; 1, 2)  $I = 10$  and 3, 4) 8 W/cm<sup>2</sup>; b) ( $l = 1.2$  mm and  $\alpha = 40$  cm<sup>-1</sup>): 3, 6) diffusion mechanism of mass transfer; 1, 4)  $m = m_0$ ; 2, 5) models of laminar mass transfer; 1-3)  $I = 1.5$  W/cm<sup>2</sup> and  $f = 5$  Hz and 4-6) 3 W/cm<sup>2</sup> and 10 Hz.

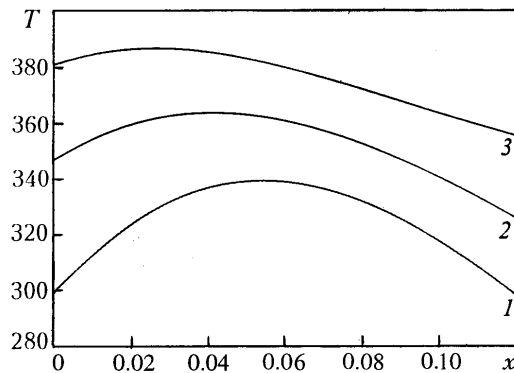


Fig. 3. Temperature vs. coordinate after the completion of irradiation for different mechanisms of mass transfer ( $t_i = 6$  sec,  $\alpha = 10$  cm<sup>-1</sup>,  $f = 5$  Hz,  $E = 2.5$  J/cm<sup>2</sup>, and  $l = 1.2$  mm): 1)  $m = m_0$ ; 2) laminar flow; 3) molecular diffusion.

ence on the temperature (3) at  $E_a = 13.5$  kJ/mole and  $D_0 = 7.2 \cdot 10^{-3}$  cm<sup>2</sup>/sec for the effective diffusion coefficient in laminar flow [12].

In this work, we have considered the following values of the variable parameters:  $\alpha = 3-40$  cm<sup>-1</sup>,  $f = 5$  and 10 Hz,  $l = 0.12-0.15$  cm,  $E = 0.2-7$  J/cm<sup>2</sup>, and  $t_i = 6-12$  sec.

**Basic Results of the Calculations.** Figure 2 shows the manner in which the surface temperature changes with time for different values of the energy density, the absorption coefficient, and two different models of mass transfer. The dependence of the temperature on the coordinate is presented in Fig. 3, whence it is seen that the maximum temperature is attained deep in the sample. The temperature at the point of maximum also depends on the energy density, the absorption coefficients, and the mass-transfer models (Fig. 4).

In [1, 2], it has been shown that heating of cartilaginous tissue above 70°C causes a sharp increase in the mobility of individual segments of the structure of the cartilaginous matrix (for example, proteoglycans or their parts), which leads to the relaxation of mechanical stresses and enables one to purposefully change the shape of the cartilages. Therefore, the time  $t_2$  in which the temperature exceeds 70°C is an important characteristic of the process. The values of  $t_2$  that correspond to the diffusion mechanism of mass transfer and in the model disregarding water motion ( $m = m_0$ ) are given in Table 1.

For comparatively small coefficients of absorption (5-10 cm<sup>-1</sup>), allowance for the diffusion mass transfer of water increases the time  $t_2$  by approximately 20% ( $\pm 5\%$ ) relative to the case  $m = m_0$ . For comparatively large coefficients of absorption ( $\sim 40$  cm<sup>-1</sup>), there can be such irradiation parameters where the quantity  $t_2$  strongly depends on

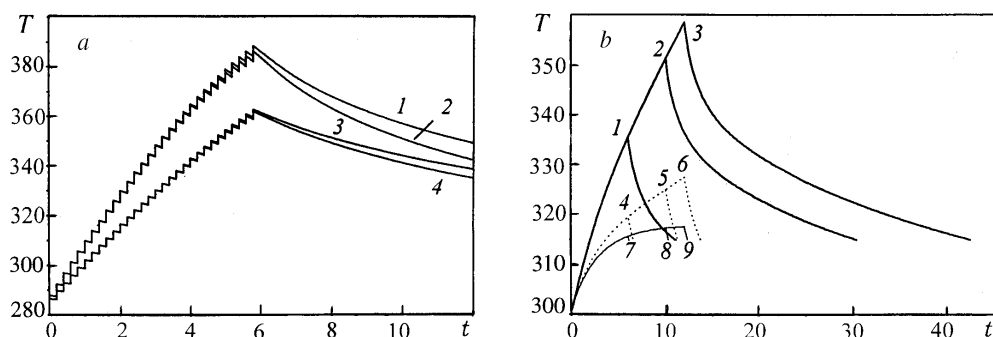


Fig. 4. Time dependence of the temperature maximum inside the sample of cartilaginous tissue for different mechanisms of mass transfer ( $l = 1.2$  mm): a) ( $\alpha = 3$  cm<sup>-1</sup> and  $f = 5$  Hz): 1, 3) diffusion mechanism of mass transfer; 2, 4)  $m = m_0$ ; 1, 2)  $I = 35$  and 3, 4)  $25$  W/cm<sup>2</sup>; b) ( $\alpha = 40$  cm<sup>-1</sup> and  $f = 10$  Hz): 1–3) diffusion mechanism of mass transfer; 4–6)  $m = m_0$ ; 7–9) law of laminar flow; 1, 4, 7)  $t_i = 6$ ; 2, 5, 8)  $10$ ; 3, 6, 9)  $12$  sec.

TABLE 1. Values of  $t_2$  for Two Mechanisms of Mass Transfer ( $l = 0.12$  cm)

$\alpha$	$t_i$	$f_r$	$E$	$t_2$ , molecular diffusion	$t_2$ , $m = m_0$
5	6	5	2.4	2.96	2.58
10	6	5	1.2	0	0
5	6	5	3.2	7.76	6.62
10	6	5	1.6	3.72	3.28
5	6	5	4.0	10.82	8.84
10	6	5	2.0	7.42	6.40
40	10	10	0.3	2.9	0
40	12	10	0.3	6.3	0

the mechanism of mass transfer of water. For some of the parameters given in the table (see, for example, the second line) a temperature of 70°C is not attained for all the considered mechanisms of mass transfer.

The distribution of the water density over the coordinate perpendicular to the surface of the sample is shown in Fig. 5. It is clear from the curves that for the model of laminar flow the total amount of evaporated water is much larger than for the model of molecular diffusion, although the degree of drying of thin surface layers is appreciably higher for molecular diffusion. As the calculation shows, it takes a few seconds after the completion of irradiation for this thin surface layer to be partially filled with water again.

The higher the concentration of water on the surface, the stronger the evaporation of it and the larger the distance of the point of temperature maximum from the irradiated surface. If we disregard the evaporation of water, the temperature maximum lies on the surface. With allowance for the evaporation, it is at a somewhat larger distance from the surface for the molecular diffusion, somewhat farther for the laminar water flow, and still farther for a constant concentration of water ( $m = m_0$ ) (Fig. 5). We have calculated the value of  $X$  (maximum value of the difference of the coordinates of the point of maximum in variation of the absorption coefficients from 5 to 20 cm<sup>-1</sup> and the sample thickness from 0.1 to 0.2 cm). The characteristic value of the difference  $X$  is in the interval 96–396 μm.

We can compare the rates of change of the temperature at the point of maximum of the temperature field for  $m = m_0$  for the models of laminar and molecular-diffusion water flow in cartilaginous tissue. As long as the temperature rises, the derivative  $\partial T/\partial t$  has comparatively high but similar (in the cases in question) values at  $t < t_i$ . After the cessation of the laser irradiation of the sample, the period of cooling begins where the rate of change of the temperature is much lower than that during the irradiation. With neglect of the water motion, the cooling rate is 1.3–1.6 times lower than in the model of laminar water flow. As far as the diffusion model is concerned, the rate of rise of the temperature is much higher for this case than for  $m = m_0$ . The rates of change of the temperature are significantly differ-

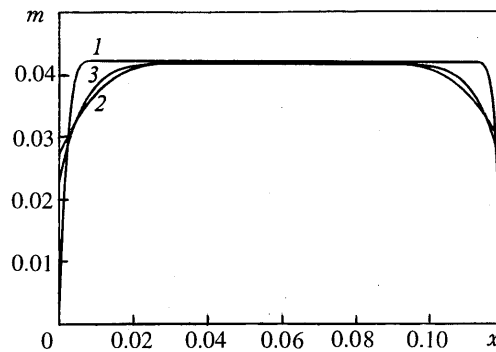


Fig. 5. Concentration of water vs. coordinate for different absorption coefficients and mechanisms of mass transfer: 1) molecular diffusion,  $t_i = 6$  sec, immediately after the completion of irradiation; 2) laminar flow,  $\alpha = 40 \text{ cm}^{-1}$ ,  $f = 5 \text{ Hz}$ ,  $E = 0.3 \text{ J/cm}^2$ ,  $l = 1.2 \text{ mm}$ ,  $t_i = 6$  sec, immediately after the completion of irradiation; 3) laminar flow,  $\alpha = 3 \text{ cm}^{-1}$ ,  $f = 5 \text{ Hz}$ ,  $E = 0.7 \text{ J/cm}^2$ ,  $l = 1.2 \text{ mm}$ ,  $t_i = 6$  sec, within 6 sec after the completion of irradiation.

ent for comparatively large absorption coefficients ( $20\text{--}40 \text{ cm}^{-1}$ ); conversely, for low values of the absorption coefficients the difference in the values of the derivatives is not so appreciable.

**Discussion of the Calculation Results.** Let us consider the influence of variable parameters (absorption coefficients, laser-pulse repetition frequencies, energy densities, and characteristic dimension of the pores in the biomaterial) on the basic characteristics of the temperature field and the concentration fields of water.

In [2], it has been shown that with allowance for the evaporation of water from the biotissue the position of the point of temperature maximum gradually moves from the irradiated surface to a point with coordinates  $W/\alpha$  ( $W$  is a number which is close to unity in order of magnitude). Since the sample is cooled from the surface, we can expect a strong difference between two cases — those of low and high values of the parameter  $Z = \alpha^2 \chi t_i$ : if  $Z < 1$ , the mass transfer of water does not have a very strong effect on the temperature maximum (Fig. 4a). Otherwise (when  $Z \gg 1$ ), allowance for the mass transfer significantly changes the value of the temperature maximum (Fig. 4b). In [2], we considered a model in which the concentration of water on the biotissue surface was constant. Of the three models considered in the present work and in [2], the temperature at the point of maximum (Fig. 4) grows most rapidly in the case of the diffusion mechanism of transfer of water and appreciably more slowly within the framework of the model of laminar flow; for the case  $m = m_0$ , the temperature maximum reaches a quasistationary value.

The results presented in Fig. 2 can easily be explained. If there is no mass transfer, the surface temperature rises more slowly than in other cases since the rate of cooling of the surface here is maximum. Within the framework of the model of laminar flow, the concentration of water on the tissue surface is somewhat lower than for  $m = m_0$  (it is limited by the inflow of the liquid from the internal layers); accordingly, the surface temperature grows somewhat more rapidly. Molecular diffusion can ensure a substantially smaller inflow of water to the surface than laminar flow; therefore, the surface temperature grows appreciably more rapidly. This is attributed to the fact that, although the quantity  $D$  increases with temperature more rapidly for the molecular diffusion than for the laminar flow, for the temperature interval in question ( $20 < T < 100^\circ\text{C}$ ) the coefficient of molecular diffusion is substantially smaller than the effective coefficient of diffusion of laminar flow. In the model of laminar flow, the effective diffusion coefficient is the larger, the larger the characteristic dimension of the pores ( $D_{\text{ef}} \sim a^2$ ). For rather large  $a$  the quantity  $D_{\text{ef}}$  tends to the coefficient of self-diffusion of water. Consequently, the larger the pore dimension, the closer the character of solution to the solutions of [2].

The larger the absorption coefficient, the more important the role of the boundary conditions for the entire temperature field. Most of the energy is released near the irradiated surface of the tissue, at a depth of about  $1/\alpha$ . The maximum of the surface temperature is always attained by the end of the irradiation period (see Fig. 2). According to [2], the stationary temperature is attained on the surface over a period of about  $2/(\alpha^2 \chi)$  in the case  $m = m_0$ . Further irradiation virtually does not change this temperature. With allowance for the motion of water in the tissue, the surface temperature (according to the calculations) is a quasistationary quantity growing with decrease in the average coeffi-

cient. Nonetheless, in this case, too, the stationary temperature is attained over a period of about  $2/(\alpha^2\chi)$ , and further irradiation changes it comparatively little. In the case of a very long irradiation, if  $2 > 2/(\alpha^2 D)$  the surface temperature smoothly changes the quasistationary value when the water in the bulk of the sample begins to be exhausted.

In considering the energy parameters of laser radiation (energy density and pulse repetition frequency), the density of the irradiation-energy intensity  $I = f_r E$  is of importance (for the considered values of the parameters). The larger the quantity  $I$ , all other parameters being equal, the higher the quasistationary surface temperature. The analogous monotonically increasing dependence relates  $I$  to the temperature at the point of maximum.

In [2], it has been shown with the example of cartilaginous tissue that the temperature maximum is below the irradiated surface and it substantially depends on the surface evaporation of water. Allowance for the mass transfer of water preserves this situation qualitatively but the numerical parameters of the thermal field change. The temperature maximum somewhat shifts toward the irradiated surface, although it does not reach it, as in the case where the evaporation of water was totally disregarded. Since the relation  $\chi \gg D$  holds, the changes in the temperature field have two characteristic times: one is directly related to heat transfer while the other, substantially longer, is related to the mass transfer of water to the sample surface. With allowance for the mass transfer of water, there can be only the quasistationary solution for the characteristic times  $\alpha^{-2} \ll \chi t$  and  $\alpha^{-2} \gg Dt$ .

In closing, we discuss selection of the parameters which describe the laminar flow of water. In [11], the structure of cartilaginous tissue is presented as a set of water-filled tubes of constant diameter  $\alpha$ ; the total volume of the pores are equal to the volume fraction of water in the biotissue ( $\approx 80\%$ ). It is assumed that only half the tubes is directed so as to ensure water flow in the direction which is of interest to us. Undoubtedly, the cartilaginous-tissue structure is substantially more complex than this model. On the basis of the spectral dependence of the scattering coefficient (it has been denoted by  $S(\lambda)$  in [13]), we can show that for  $a \sim 0.1\text{--}2\ \mu\text{m}$  the cartilage structure has a fractal character. Since the laminar liquid flow in fractal structures has not been investigated, we confine ourselves to the model of [11]. The rate of water flowing in a cylindrical channel is in proportion to  $q \sim a^4/\mu$ . The total volume of the pores is fixed; therefore, the flow of water per unit area is in proportion to the effective coefficient of diffusion  $D_{\text{ef}} \sim a^2/\mu$ . At room temperature, the effective coefficient of diffusion  $D_{\text{ef}}$  related to the laminar flow is 45 times smaller than the coefficient of molecular diffusion. Therefore, the contribution of the latter is small when the water flow is considered. Assuming that  $\mu = 1.0\ \text{MPa}\cdot\text{sec}$  (this is the value of the dynamic viscosity of pure water at  $T = 25^\circ\text{C}$  [12]), we obtain the estimate of the effective hydrodynamic tube diameter  $a \approx 56 \cdot 10^{-8}\ \text{cm}$  [11]. This estimate of the pore dimension is nearly an order of magnitude lower than the results of the measurements [14] performed using an atomic-force microscope, where it has been shown that the distribution of the diameters of the cartilaginous-tissue pores is in the range of  $a \approx (200\text{--}600) \cdot 10^{-8}\ \text{cm}$ . In our opinion, the difference in the estimates of this quantity is attributed to the structure itself of the cartilaginous tissue the pores inside which are not smooth channels. The walls themselves of these channels are sustained by the dipole-dipole interaction of proteoglycans. The polar water molecules flowing in such "channels with obstacles" interact with the proteoglycans, which retards their motion.

The viscosity coefficient for pure water  $\mu$  decreases twice on heating from 20 to  $55^\circ\text{C}$ ; for this reason we assume that for the laminar flow the effective diffusion coefficient depends on the temperature. In the temperature range  $0\text{--}100^\circ\text{C}$ , the viscosity of water is described by the Arrhenius dependence with the activation energy given above [12]. In actual practice, water in cartilaginous tissue represents a solution of different substances, and accordingly its dynamic viscosity can strongly differ from the values for pure water. However, within the framework of the model calculation, when the temperature dependence of the viscosity of water with substances dissolved in it is unknown we have to confine ourselves to the data given in [12].

## CONCLUSIONS

We have investigated the temperature field in cartilaginous tissue in the case of the action of IR-radiation pulses on it. We have considered two possible models of mass transfer of water in the cartilaginous tissue (molecular diffusion and the model of laminar flow) and have calculated the effective values of the diffusion coefficients for them. It has been shown that the amount of evaporated water is substantially larger in the case of the laminar water flow in the sample, where the effective diffusion coefficient is relatively large as compared to the molecular-diffusion mechanism of mass transfer, for which  $D$  is substantially smaller, although in the latter case the concentration of water

in the surface layer is appreciably lower than for the model of laminar flow. For the laminar flow the depth of a partially dried layer is appreciably larger than for the case of molecular diffusion. If  $\alpha^2\chi t_i \gg 1$  and the density of the energy intensity  $I$  is constant, the temperature at the point of maximum increases most rapidly in the case of the diffusion mechanism of transfer of water and much more slowly in laminar flow, while for a constant concentration of water the temperature maximum reaches a constant (quasistationary) value comparatively rapidly. A similar relation is observed for the irradiated-surface temperature if  $\alpha^2\chi t_i \gg 1$ . The position of the point of maximum is close to the value of  $(1/\alpha)$ . The characteristic distance between the points of maximum obtained with the use of different models is in the interval 96–396  $\mu\text{m}$  (absorption coefficients of 5 to 20  $\text{cm}^{-1}$ , sample thickness of 1 to 2 mm). As long as the temperature rises, at  $t > t_i$  the derivative  $\partial T/\partial t$  has comparatively high but similar (within the framework of different models) values. The larger  $I$ , the larger the derivative  $\partial T/\partial t$ . The derivative is a positive quantity if not constant, and  $\partial^2 T/\partial t \partial I > 0$ . Conversely, for a constant concentration of water the cooling rate is 1.3–1.6 times higher than that in the model of laminar water flow. The larger the absorption coefficient, the larger the portion of laser energy expended on evaporating water.

This work was carried out with assistance from the Russian Foundation for Basic Research (grant 02-02-16246).

## NOTATION

$T = T(x, t)$ , temperature,  $^{\circ}\text{C}$  and  $\text{K}$ ;  $T_0$ , initial temperature,  $^{\circ}\text{C}$  and  $\text{K}$ ;  $T^*$ , minimum temperature of denaturation and relaxation of stresses in cartilages,  $^{\circ}\text{C}$  and  $\text{K}$ ;  $m = m(x, t)$ , concentration of water in the biotissue,  $\text{mole}/\text{cm}^3$ ;  $m_0$ , initial concentration of water,  $\text{mole}/\text{cm}^3$ ;  $x$ , coordinate perpendicular to the sample surface,  $\text{cm}$ ;  $x = 0$ , coordinate of the irradiated surface;  $l$ , thickness of the sample,  $\text{cm}$ ;  $t$ , running time,  $\text{sec}$ ;  $X$ , value of the difference of the coordinates of the point of temperature maximum in variation of different parameters,  $\text{cm}$ ;  $h$ , root-mean-square distance traversed by the water molecules,  $\text{cm}$ ;  $t = 0$ , instant of the beginning of irradiation;  $t_0$ , calculation time,  $\text{sec}$ ;  $t_i$ , duration of laser irradiation,  $\text{sec}$ ;  $\tau_p$ , laser-pulse duration,  $\text{sec}$ ;  $t_2$ , time in which  $T > T^*$  (subscript 2 corresponds to the "second" stage in laser irradiation of the sample, i.e., to the period of the denaturation reaction);  $D = D(T)$ , effective coefficient of diffusion of water in the biotissue,  $\text{cm}^2/\text{sec}$  (it is denoted by  $D_{\text{ef}}$  for the laminar-flow model);  $D_0$ , preexponential factor of the diffusion coefficient,  $\text{cm}^2/\text{sec}$ ;  $E_a$ , diffusion activation energy,  $\text{J}/\text{mole}$ ;  $R$ , universal gas constant;  $C_p$ , specific (in terms of volume) heat of the biotissue,  $\text{J}/(\text{m}^3 \cdot \text{K})$ ;  $\lambda$ , thermal conductivity,  $\text{W}/(\text{m} \cdot \text{K})$ ;  $\chi$ , thermal diffusivity,  $\text{cm}^2/\text{sec}$ ;  $Q$ , heat of evaporation of water,  $\text{J}/\text{mole}$ ;  $Q_v$ ,  $Q/m_0$ , heat of evaporation per unit volume,  $\text{J}/\text{cm}^3$ ;  $\nu$ , frequency of intermolecular vibrations of the water molecules,  $\text{Hz}$ ;  $\alpha$ , coefficient of absorption of laser radiation,  $\text{cm}^{-1}$ ;  $f_r$ , laser-pulse repetition frequency,  $\text{Hz}$ ;  $E$ , density of the laser-pulse energy,  $\text{J}/\text{cm}^2$ ;  $I = Ef_r$ , density of the laser-radiation intensity,  $\text{W}/\text{cm}^2$ ;  $q$ , flow rate of water per unit time,  $\text{mole}/(\text{cm}^2 \cdot \text{sec})$ ;  $\Delta P$ , difference of the pressures of water between the surfaces of the plate,  $\text{dyn}/\text{cm}^2$ ;  $A$ , part of the sample area occupied by the pores;  $\mu$ , dynamic viscosity of water,  $\text{MPa} \cdot \text{sec}$ ;  $\nu_s$ , velocity of sound in the medium,  $\text{m}/\text{sec}$ ;  $\rho_w$ , specific density of water,  $\text{g}/\text{cm}^3$ ;  $a$ , characteristic dimension of a pore,  $\text{cm}$ ; function  $\delta(x) = 1$  for  $x = 0$ ,  $\delta(x) = -1$  for  $x = l$ , and  $\delta(x) = 0$  for the remaining values of  $x$ ;  $S(\lambda)$ , spectral dependence of the coefficient of scattering of the biotissue,  $\text{cm}^{-1}$ . Subscripts: i, irradiation; p, pulse; ef, effective; a, activation; v, volume; s, sound; w, water; r, repetition;  $\rho$ , density.

## REFERENCES

1. E. Sobol' (Sobol), A. Sviridov, A. Omel'chenko, V. Bagratashvili, M. Kitai, S. E. Harding, N. Jones, K. Jumel, M. Mertig, W. Pompe, Yu. Ovchinnikov, A. Shekhter, and V. Svistushkin, *Biotechnol. Genetic Eng. Rev.*, **17**, 539–564 (2000).
2. E. N. Sobol' (Sobol), M. S. Kitai, N. Jones, A. P. Sviridov, T. Milner, and B. Wong, *IEEE J. Quant. Electronics*, **35**, 532–540 (1999).
3. E. Sobol' (Sobol), in: *Phase Transformations and Ablation in Laser-Treated Solids*, Wiley, New York (1995), p. 4.
4. L. D. Landau and E. M. Lifshits, *Hydrodynamics* [in Russian], Ch. 2, Moscow (1986).
5. A. V. Luikov, *Heat and Mass Transfer: Handbook* [in Russian], Moscow (1972).



6. T. Kumada, in: N. P. Cheremisinoff (ed.), *Handbook of Heat and Mass Transfer*, Vol. 1, New York (1992), pp. 231–250.
7. A. A. Samarskii, *The Theory of Difference Schemes* [in Russian], Moscow (1977).
8. W. R. Wasow and G. E. Forsythe, *Finite-Difference Methods for Partial Differential Equations* [Russian translation], Moscow (1963).
9. V. N. Bagratashvili, E. N. Sobol' (Sobol), A. P. Sviridov, V. K. Popov, A. I. Omel'chenko, and S. M. Howdle, *J. Biomechanics*, **30**, No. 8, 813–817 (1997).
10. R. Knauss, J. Schiller, G. Fleischer, J. Karger, and K. Arnold, *Magn. Res. Medicine*, **41**, 285–299 (1999).
11. V. C. Mow, M. H. Holmes, W. M. and Law, *J. Biomechanics*, **17**, No. 5, 377–394 (1984).
12. I. T. Goronovskii, Yu. P. Nazarenko, and E. F. Nekryach, *Concise Handbook on Chemistry* [in Russian], Kiev (1987).
13. N. V. Bagratashvili, A. P. Sviridov, E. N. Sobol' (Sobol), and M. S. Kitai, *Proc. SPIE*, **3254**, 398–408 (1997).
14. E. Sobol' (Sobol), A. Omel'chenko, M. Mertig, and W. Pompe, *Lasers Med. Sci.*, **15**, 15–23 (2000).